

Question 1

Let $a_n = \frac{3n-1}{n+2}$ for $n \geq 1$.

- (a) Find $L = \lim_{n \rightarrow \infty} a_n$.
- (b) Solve $|a_n - L| < 0.01$ exactly and determine the smallest integer N such that the inequality holds for every $n \geq N$.

Question 1 – Solution**Step 1: Find the candidate limit.**

The numerator and denominator have the same degree. Divide both by n :

$$a_n = \frac{3 - 1/n}{1 + 2/n} \longrightarrow \frac{3}{1} = 3.$$

Since $1/n \rightarrow 0$ and $2/n \rightarrow 0$, the quotient law gives $L = 3$.

Step 2: Express the error exactly.

Rather than approximate numerically, subtract the limit and simplify:

$$|a_n - 3| = \left| \frac{3n - 1}{n + 2} - 3 \right| = \frac{7}{n + 2}.$$

Step 3: Solve the strict accuracy inequality.

Therefore

$$\frac{7}{n + 2} < 0.01 = \frac{1}{100} \iff n > 698.$$

Thus every integer $n \geq 699$ satisfies the requirement.

Step 4: Verify minimality.

The smallest possible integer is $N = 699$. At the preceding index, $n = 698$, the error is $7/700 = 0.01$, which equals the tolerance and is not strictly less than it. Hence no smaller N works.