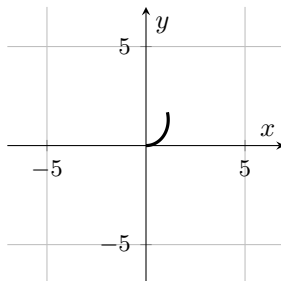
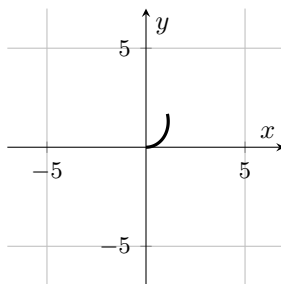


Question 10**Problem**

Find a so that $r = a\theta$, $0 \leq \theta \leq 1$, has length $\sqrt{2} + \operatorname{arsinh} 1$.



Question 10 – Solution**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

Simplify the expression under the square root before evaluating or reporting the integral.

3. Its length is $|a| \int_0^1 \sqrt{1 + \theta^2} d\theta = \frac{|a|}{2}(\sqrt{2} + \operatorname{arsinh} 1)$.
4. Thus $\boxed{a = \pm 2}$.