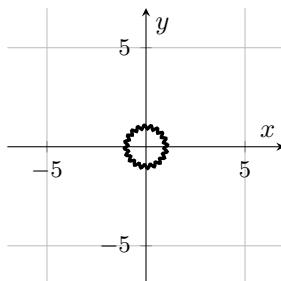
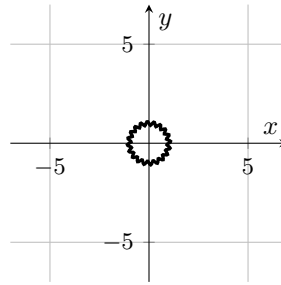


Question 6**Problem**

Without evaluating, compare the lengths of $r = 1$ and $r = 1 + 0.1 \sin(20\theta)$ over $0 \leq \theta \leq 2\pi$.



Question 6 – Solution

**Solution**

1. Differentiate the polar radius to obtain $r' = dr/d\theta$ and choose an interval that traces the requested arc exactly once.
2. Use the polar arc-length formula

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

Simplify the expression under the square root before evaluating or reporting the integral.

3. Here $r > 0$ and $r' = 2 \cos 2\theta$. Thus $\sqrt{r^2 + (r')^2} \geq r$, with strict inequality except at finitely many angles. Consequently $L > \int_0^{2\pi} (1 + 0.1 \sin 2\theta) d\theta = 2\pi$, the unit circle's length.