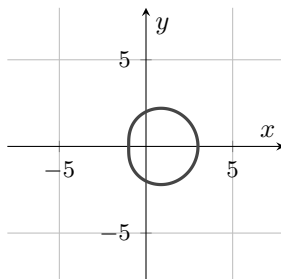
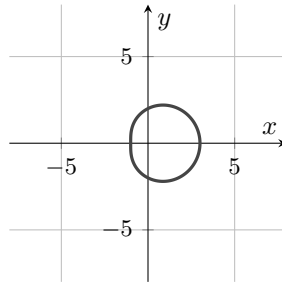


Question 8**Problem**

Two polar curves satisfy $0 \leq r_1(\theta) \leq r_2(\theta)$. Explain why the area between them involves squares, not merely $r_2 - r_1$.



Question 8 – Solution**Solution**

1. Determine an interval of angles that sweeps out the requested region exactly once, using symmetry when it simplifies the work.
2. For a polar boundary $r = f(\theta)$, use

$$A = \frac{1}{2} \int_a^b r^2 d\theta.$$

For a region between two curves, subtract the inner squared radius from the outer squared radius before integrating.

3. A thin polar sector has area $\frac{1}{2}r^2 d\theta$.
4. Subtracting sector areas yields $\frac{1}{2}(r_2^2 - r_1^2)d\theta$, reflecting increasing width with radius.