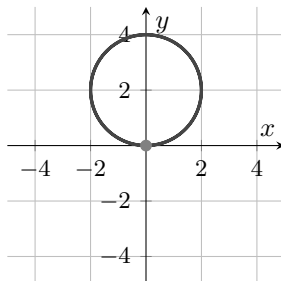
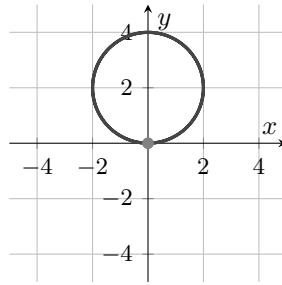


Question 10**Problem**

For the circle $r = 4 \sin \theta$, find the tangent at its highest point using polar calculus.



Question 10 – Solution

**Solution**

1. Write the polar curve parametrically as

$$x(\theta) = r(\theta) \cos \theta, \quad y(\theta) = r(\theta) \sin \theta.$$

Differentiation gives

$$\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta, \quad \frac{dy}{d\theta} = r' \sin \theta + r \cos \theta.$$

2. Wherever $dx/d\theta \neq 0$, compute

$$\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}.$$

Test numerator and denominator separately when locating horizontal or vertical tangents.

3. The highest point occurs at $\theta = \pi/2$, giving $(0, 4)$.
4. The slope numerator is 0 and denominator -4 , so the tangent is horizontal: $y = 4$.