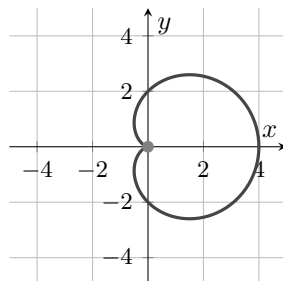


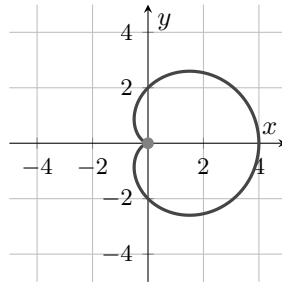
Question 8

Problem

Find a value of a so that $r = a + 2 \cos \theta$ has tangent slope 1 at $\theta = \pi/2$.



Question 8 – Solution



Solution

1. Write the polar curve parametrically as

$$x(\theta) = r(\theta) \cos \theta, \quad y(\theta) = r(\theta) \sin \theta.$$

Differentiation gives

$$\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta, \quad \frac{dy}{d\theta} = r' \sin \theta + r \cos \theta.$$

2. Wherever $dx/d\theta \neq 0$, compute

$$\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}.$$

Test numerator and denominator separately when locating horizontal or vertical tangents.

3. At $\pi/2$, the slope is $2/a$.

4. Setting $2/a = 1$ gives $\boxed{a = 2}$.