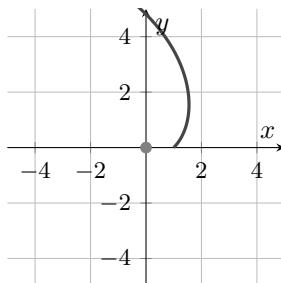


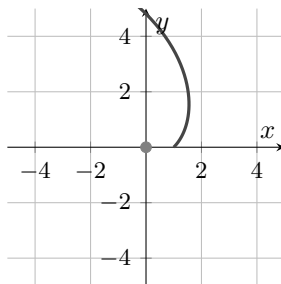
Question 7

Problem

For $r = e^\theta$, show that the angle between the radius vector and tangent is constant.



Question 7 – Solution



Solution

1. Write the polar curve parametrically as

$$x(\theta) = r(\theta) \cos \theta, \quad y(\theta) = r(\theta) \sin \theta.$$

Differentiation gives

$$\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta, \quad \frac{dy}{d\theta} = r' \sin \theta + r \cos \theta.$$

2. Wherever $dx/d\theta \neq 0$, compute

$$\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}.$$

Test numerator and denominator separately when locating horizontal or vertical tangents.

3. In polar components, tangent has radial component $r' = r$ and transverse component r .
4. Thus it makes angle $\arctan(r/r) = \boxed{\pi/4}$ with the radius vector.