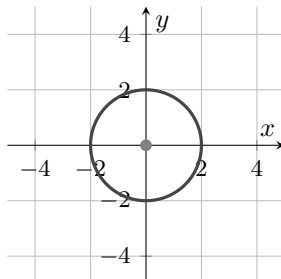
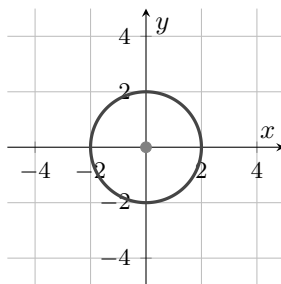


Question 6**Problem**

At an intersection of $r = 2$ and $r = 4 \cos \theta$ in the upper half-plane, find the acute angle between the two curves.



Question 6 – Solution



Solution

1. Write the polar curve parametrically as

$$x(\theta) = r(\theta) \cos \theta, \quad y(\theta) = r(\theta) \sin \theta.$$

Differentiation gives

$$\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta, \quad \frac{dy}{d\theta} = r' \sin \theta + r \cos \theta.$$

2. Wherever $dx/d\theta \neq 0$, compute

$$\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}.$$

Test numerator and denominator separately when locating horizontal or vertical tangents.

3. At $\theta = \pi/3$, the circle $r = 2$ has tangent direction $5\pi/6$.
4. The second circle has slope $1/\sqrt{3}$, hence tangent direction $\pi/6$.
5. The smaller angle between the tangent lines is $\boxed{\pi/3}$.