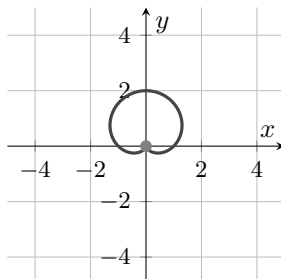
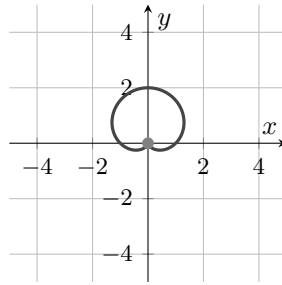


Question 3**Problem**

A student claims that $\theta = 0$ gives the rightmost point and a vertical tangent of $r = 1 + \sin \theta$. Test both parts of the claim.



Question 3 – Solution



Solution

1. Write the polar curve parametrically as

$$x(\theta) = r(\theta) \cos \theta, \quad y(\theta) = r(\theta) \sin \theta.$$

Differentiation gives

$$\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta, \quad \frac{dy}{d\theta} = r' \sin \theta + r \cos \theta.$$

2. Wherever $dx/d\theta \neq 0$, compute

$$\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}.$$

Test numerator and denominator separately when locating horizontal or vertical tangents.

3. At $\theta = 0$, the point is $(1, 0)$ and the slope is 1, so the tangent is not vertical.
4. Also $x'(0) = 1 > 0$, so x is still increasing; the point is not rightmost.
5. Both parts are false.