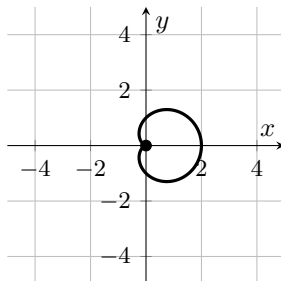


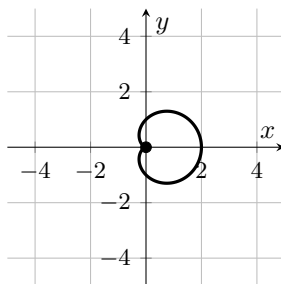
Question 2

Problem

Find all horizontal tangents of the cardioid $r = 1 + \cos \theta$ away from the pole.



Question 2 – Solution



Solution

1. Write the polar curve parametrically as

$$x(\theta) = r(\theta) \cos \theta, \quad y(\theta) = r(\theta) \sin \theta.$$

Differentiation gives

$$\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta, \quad \frac{dy}{d\theta} = r' \sin \theta + r \cos \theta.$$

2. Wherever $dx/d\theta \neq 0$, compute

$$\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}.$$

Test numerator and denominator separately when locating horizontal or vertical tangents.

3. The numerator $r' \sin \theta + r \cos \theta = \cos \theta + \cos 2\theta$ vanishes.
4. This gives $\cos \theta = 1/2$ or -1 , and excluding the pole leaves $\theta = \pi/3, 5\pi/3$. At these angles $dx/d\theta \neq 0$, and the points are $(3/4, \pm 3\sqrt{3}/4)$. The tangent lines are $y = \pm 3\sqrt{3}/4$.