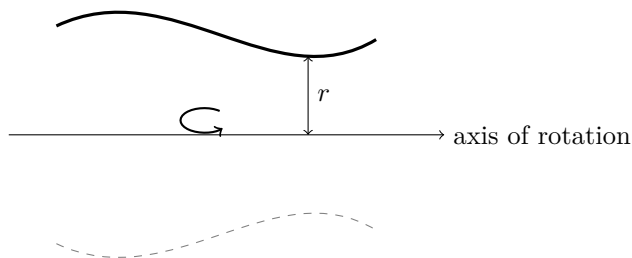


Calculus II Section 3-5: Surface Area with Parametric Equations

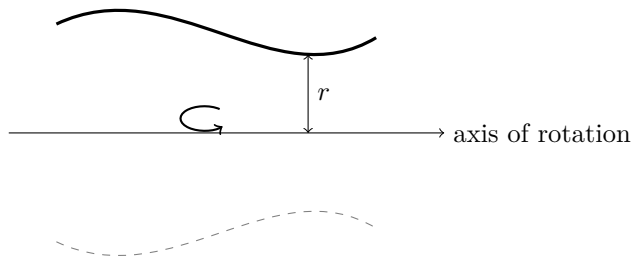
Question 10

Problem

Two curves have the same length and the same arc-length-weighted average distance from an axis, and each sweeps its surface exactly once. Must their surfaces of revolution have equal area?



Question 10 – Solution



Solution

- Let $r(s)$ denote the nonnegative distance from a point on a generating curve to the axis, where s is arc length. Its arc-length-weighted average distance is

$$r_{\text{avg}} = \frac{1}{L} \int_C r \, ds.$$

- Multiply by L :

$$\int_C r \, ds = L r_{\text{avg}}.$$

The surface area of revolution is therefore

$$S = 2\pi \int_C r \, ds = 2\pi L r_{\text{avg}}.$$

- If two curves have the same length L and the same arc-length-weighted average radius, this formula gives the same value of S for both.
- Thus the answer is yes, provided each generating curve sweeps its surface exactly once. If a parametrization retraces the curve or distinct pieces generate the same surface, the integral counts multiplicity and the comparison requires removing that double-counting first.