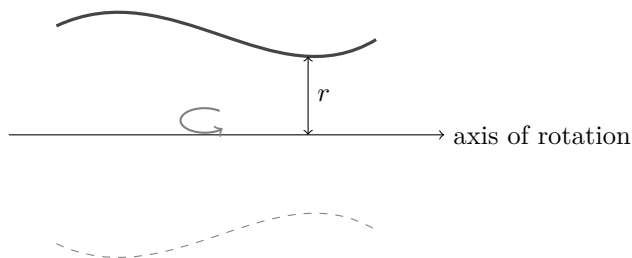


Calculus II Section 3-5: Surface Area with Parametric Equations

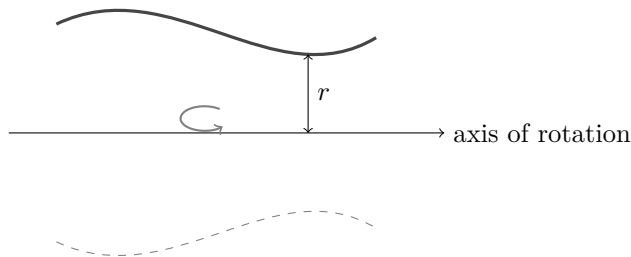
Question 9

Problem

Rotate the astroid quadrant $x = \cos^3 t, y = \sin^3 t, 0 \leq t \leq \pi/2$, about the x -axis. Set up a simplified integral.



Question 9 – Solution



Solution

1. Differentiate:

$$x'(t) = -3 \cos^2 t \sin t, \quad y'(t) = 3 \sin^2 t \cos t.$$

2. As in the astroid arc-length calculation,

$$ds = 3 |\sin t \cos t| dt.$$

Both factors are nonnegative on $0 \leq t \leq \pi/2$, so

$$ds = 3 \sin t \cos t dt.$$

3. The radius about the x -axis is $y = \sin^3 t$. Therefore,

$$\begin{aligned} S &= 2\pi \int_0^{\pi/2} y ds \\ &= 6\pi \int_0^{\pi/2} \sin^4 t \cos t dt. \end{aligned}$$

4. Let $u = \sin t$, so $du = \cos t dt$ and $0 \leq u \leq 1$:

$$S = 6\pi \int_0^1 u^4 du = 6\pi \left[\frac{u^5}{5} \right]_0^1 = \boxed{\frac{6\pi}{5}}.$$