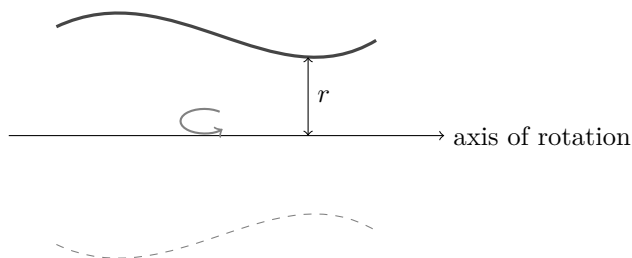


Calculus II Section 3-5: Surface Area with Parametric Equations

Question 8

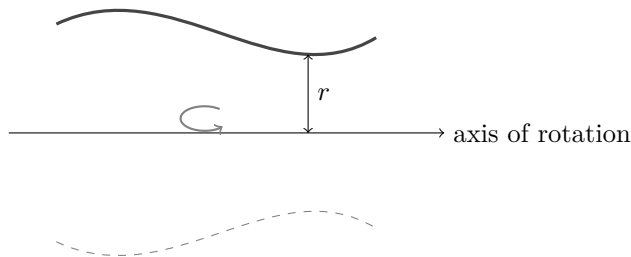
Problem

A curve crosses the axis of rotation. What precaution is needed in $S = 2\pi \int y \, ds$?



Calculus II Section 3-5: Surface Area with Parametric Equations

Question 8 – Solution



Solution

1. In $S = 2\pi \int y \, ds$, the factor multiplying 2π must be the radius of rotation. A radius is a distance and therefore cannot be negative.
2. When the axis is the x -axis, the distance from $(x(t), y(t))$ to that axis is

$$r(t) = |y(t)|,$$

not signed $y(t)$.

3. The correct general formula is therefore

$$S = 2\pi \int_a^b |y(t)| \sqrt{x'(t)^2 + y'(t)^2} \, dt.$$

Equivalently, split the integral at every parameter value where $y(t) = 0$ and use the appropriate sign on each subinterval.

4. Without the absolute value, contributions below the axis would be negative and could cancel contributions above the axis. Also check whether portions on opposite sides of the axis generate the same physical surface; if they do, restrict the interval to prevent double-counting.