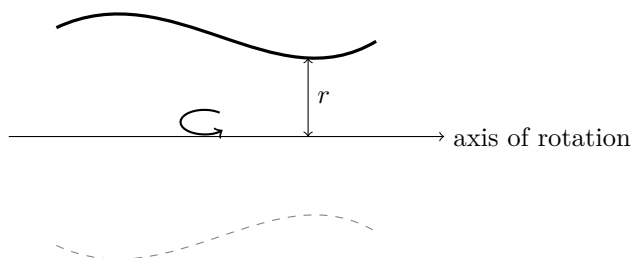


Calculus II Section 3-5: Surface Area with Parametric Equations

Question 7

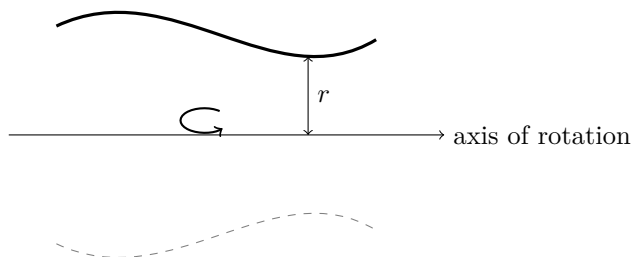
Problem

A cylinder is generated parametrically by rotating the horizontal segment $x = t, y = h, 0 \leq t \leq L$, about the x -axis, where $h, L > 0$. Derive its lateral area.



Calculus II Section 3-5: Surface Area with Parametric Equations

Question 7 – Solution



Solution

1. Differentiate the horizontal segment:

$$x'(t) = 1, \quad y'(t) = 0.$$

Hence

$$ds = \sqrt{1^2 + 0^2} dt = dt.$$

2. Every point of the segment is the constant distance h from the x -axis, assuming $h \geq 0$. Thus the radius of rotation is h .
3. Apply the surface-area formula:

$$\begin{aligned} S &= 2\pi \int_0^L h dt \\ &= 2\pi h [t]_0^L = \boxed{2\pi h L}. \end{aligned}$$

4. This is circumference times length: $(2\pi h)L$, which is precisely the lateral area of a cylinder of radius h and length L .