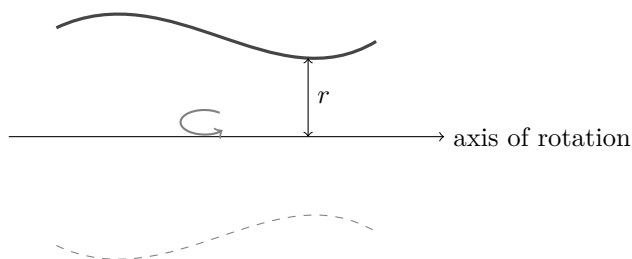


Calculus II Section 3-5: Surface Area with Parametric Equations

Question 6

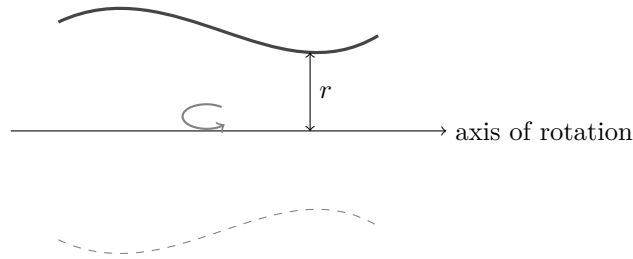
Problem

Rotate $x = e^t$, $y = e^{-t}$, $0 \leq t \leq \ln 2$, about the x -axis. Write the exact surface-area integral in simplest form.



Calculus II Section 3-5: Surface Area with Parametric Equations

Question 6 – Solution



Solution

1. Differentiate:

$$x'(t) = e^t, \quad y'(t) = -e^{-t}.$$

Therefore,

$$ds = \sqrt{e^{2t} + e^{-2t}} dt.$$

2. The curve lies above the x -axis because $y = e^{-t} > 0$, so the radius of rotation is $y = e^{-t}$.
3. Multiply the radius by ds and simplify:

$$\begin{aligned} y ds &= e^{-t} \sqrt{e^{2t} + e^{-2t}} dt \\ &= \sqrt{e^{-2t}(e^{2t} + e^{-2t})} dt \\ &= \sqrt{1 + e^{-4t}} dt. \end{aligned}$$

4. Hence the requested exact integral is

$$S = 2\pi \int_0^{\ln 2} \sqrt{1 + e^{-4t}} dt.$$