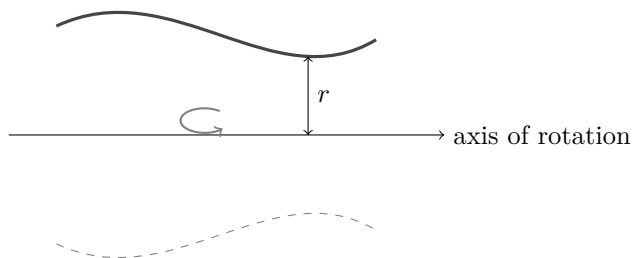


Calculus II Section 3-5: Surface Area with Parametric Equations

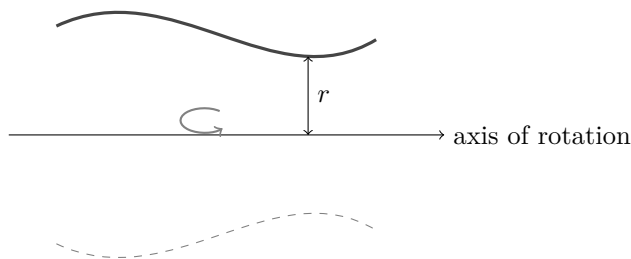
Question 5

Problem

Why is rotating $x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$, about the x -axis a bad direct use of the surface formula?



Question 5 – Solution



Solution

1. The interval $0 \leq t \leq \pi$ traces the upper semicircle, while $\pi \leq t \leq 2\pi$ traces the lower semicircle.
2. When either semicircle is rotated about the x -axis, it generates the entire unit sphere. A point (x, y) and its reflection $(x, -y)$ produce the same circle of revolution because both are the same distance $|y|$ from the axis.
3. If the full interval is used with the correct radius $|y|$, then $ds = dt$ and

$$2\pi \int_0^{2\pi} |\sin t| dt = 8\pi,$$

which is twice the unit sphere's area. The formula counts the repeated sweep.

4. If one incorrectly uses signed $y = \sin t$ instead, the upper and lower contributions cancel and give zero, which is also not physical surface area.
5. Use one generating semicircle only, for example

$$S = 2\pi \int_0^\pi \sin t dt = \boxed{4\pi}.$$

Thus the full-circle interval is unsuitable because it double-covers the surface.