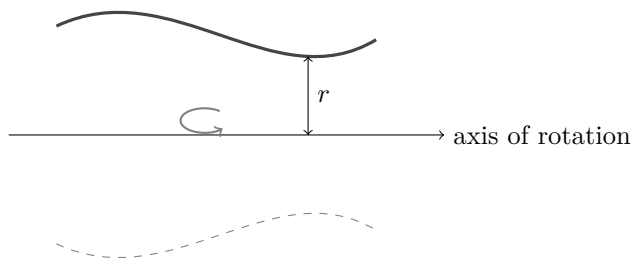


Calculus II Section 3-5: Surface Area with Parametric Equations

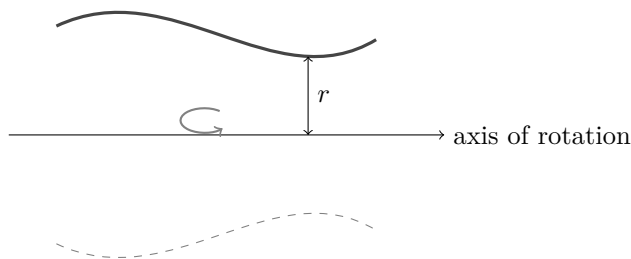
Question 4

Problem

Set up, but do not evaluate, the area formed by rotating $x = t^2$, $y = t^3$, $0 \leq t \leq 1$, about the y -axis.



Question 4 – Solution



Solution

1. Differentiate the coordinates:

$$x'(t) = 2t, \quad y'(t) = 3t^2.$$

2. Compute the arc-length element:

$$\begin{aligned} ds &= \sqrt{(2t)^2 + (3t^2)^2} dt \\ &= \sqrt{4t^2 + 9t^4} dt = |t|\sqrt{4 + 9t^2} dt. \end{aligned}$$

Since $0 \leq t \leq 1$, this becomes $ds = t\sqrt{4 + 9t^2} dt$.

3. Rotation is about the y -axis, so the radius is the distance $x = t^2$.
4. Therefore, without evaluating,

$$\begin{aligned} S &= 2\pi \int_0^1 x(t) ds \\ &= \boxed{2\pi \int_0^1 t^3 \sqrt{4 + 9t^2} dt}. \end{aligned}$$