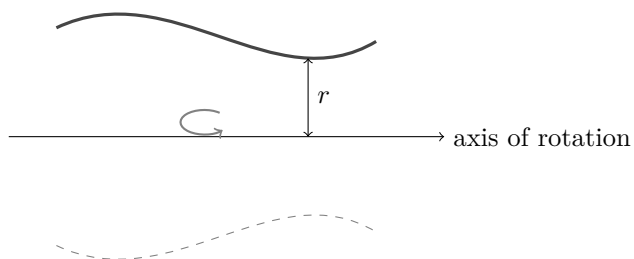


Calculus II Section 3-5: Surface Area with Parametric Equations

Question 3

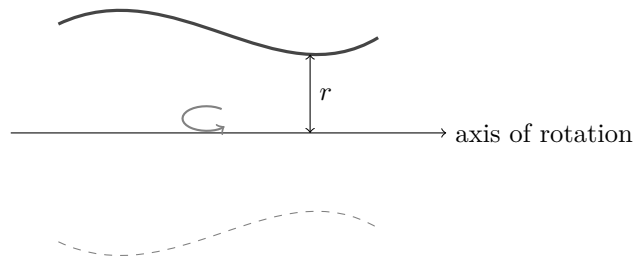
Problem

A circle $x = 3 + \cos t$, $y = \sin t$ rotates about the y -axis. Find the torus area without citing a memorized formula.



Calculus II Section 3-5: Surface Area with Parametric Equations

Question 3 – Solution



Solution

1. The interval $0 \leq t \leq 2\pi$ traces the generating circle once. Differentiate:

$$x'(t) = -\sin t, \quad y'(t) = \cos t.$$

Hence

$$ds = \sqrt{\sin^2 t + \cos^2 t} dt = dt.$$

2. The circle lies between $x = 2$ and $x = 4$, entirely to the right of the y -axis. Therefore, the radius of rotation is the nonnegative distance

$$x = 3 + \cos t.$$

3. Apply $S = 2\pi \int x ds$:

$$\begin{aligned} S &= 2\pi \int_0^{2\pi} (3 + \cos t) dt \\ &= 2\pi [3t + \sin t]_0^{2\pi} \\ &= 2\pi(6\pi) = \boxed{12\pi^2}. \end{aligned}$$