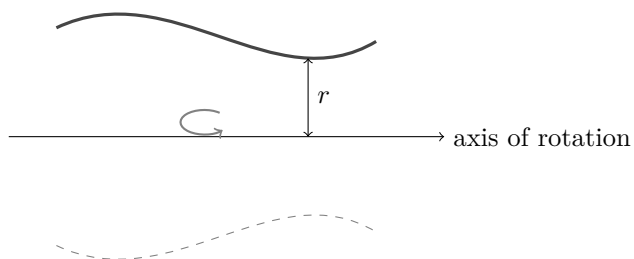


Calculus II Section 3-5: Surface Area with Parametric Equations

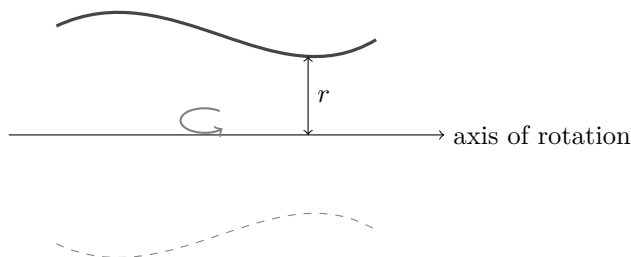
Question 2

Problem

The upper semicircle $x = R \cos t$, $y = R \sin t$, $0 \leq t \leq \pi$, rotates about the x -axis. Recover the sphere's area.



Question 2 – Solution



Solution

1. Differentiate:

$$x'(t) = -R \sin t, \quad y'(t) = R \cos t.$$

Thus

$$ds = \sqrt{R^2 \sin^2 t + R^2 \cos^2 t} dt = R dt.$$

2. On $0 \leq t \leq \pi$, the generating semicircle lies above the x -axis, so its radius of rotation is

$$y = R \sin t \geq 0.$$

This semicircle generates the sphere exactly once.

3. Substitute into $S = 2\pi \int y ds$:

$$\begin{aligned} S &= 2\pi \int_0^\pi (R \sin t)(R dt) \\ &= 2\pi R^2 [-\cos t]_0^\pi \\ &= 2\pi R^2 (2) = \boxed{4\pi R^2}. \end{aligned}$$