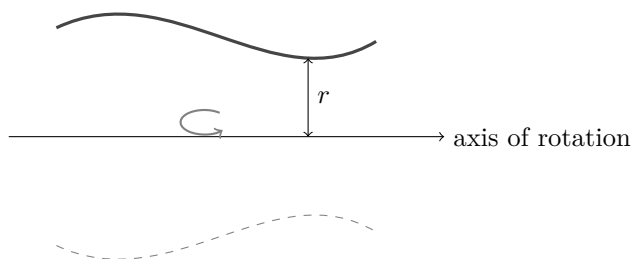


Calculus II Section 3-5: Surface Area with Parametric Equations

Question 1

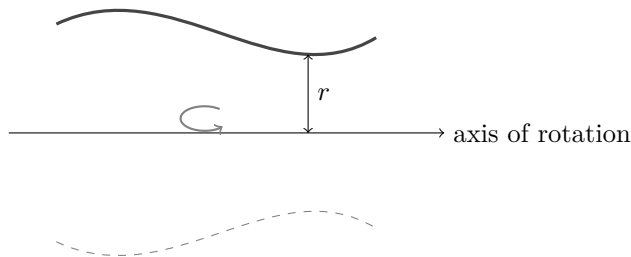
Problem

Rotate the line segment $x = t$, $y = 1 - t$, $0 \leq t \leq 1$, about the x -axis. Find the surface area and identify the surface.



Calculus II Section 3-5: Surface Area with Parametric Equations

Question 1 – Solution



Solution

1. Differentiate the coordinates:

$$x'(t) = 1, \quad y'(t) = -1.$$

Therefore,

$$ds = \sqrt{1^2 + (-1)^2} dt = \sqrt{2} dt.$$

2. The distance from (x, y) to the x -axis is $y = 1 - t$, which is nonnegative on $0 \leq t \leq 1$.
3. Apply the surface-area formula:

$$\begin{aligned} S &= 2\pi \int_0^1 y \, ds \\ &= 2\pi\sqrt{2} \int_0^1 (1-t) \, dt \\ &= 2\pi\sqrt{2} \left[t - \frac{t^2}{2} \right]_0^1 = \boxed{\pi\sqrt{2}}. \end{aligned}$$

4. The segment runs from $(0, 1)$ to $(1, 0)$. Rotating it about the x -axis produces a right circular cone with radius 1, height 1, and slant height $\sqrt{2}$. The result agrees with $\pi r \ell = \pi\sqrt{2}$.