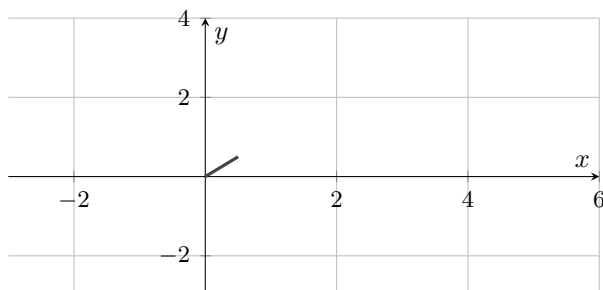


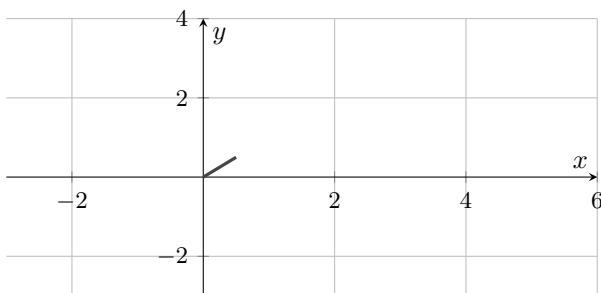
Question 10

Problem

Find $a > 0$ so that the line path $x = at$, $y = (1 - a)t$, $0 \leq t \leq 1$, has minimum length.



Question 10 – Solution



Solution

1. Differentiate the line parametrization:

$$x'(t) = a, \quad y'(t) = 1 - a.$$

The speed is constant, so

$$L(a) = \int_0^1 \sqrt{a^2 + (1-a)^2} dt = \sqrt{a^2 + (1-a)^2}.$$

2. Because the square-root function is increasing, minimizing $L(a)$ is equivalent to minimizing

$$L(a)^2 = a^2 + (1-a)^2 = 2a^2 - 2a + 1.$$

Complete the square:

$$2a^2 - 2a + 1 = 2 \left(a - \frac{1}{2} \right)^2 + \frac{1}{2}.$$

3. The squared term is smallest when $a = 1/2$, which satisfies $a > 0$. Therefore,

$$\boxed{a = \frac{1}{2}}.$$

4. At this value, the minimum length is

$$L_{\min} = \sqrt{\frac{1}{2}} = \boxed{\frac{1}{\sqrt{2}}}.$$

Geometrically, the endpoint $(a, 1-a)$ is then $(1/2, 1/2)$, the closest point on the line $x + y = 1$ to the origin.