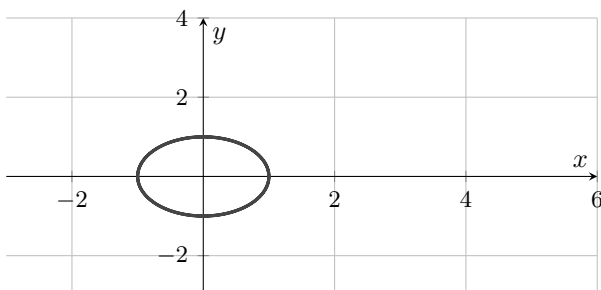


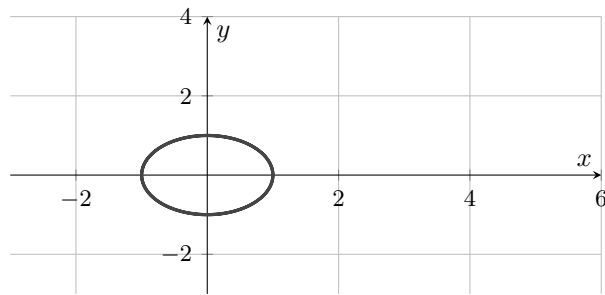
Question 9

Problem

A particle goes around the unit circle twice. Is its path length 2π or 4π ? Justify using $x = \cos 2t$, $y = \sin 2t$, $0 \leq t \leq 2\pi$.



Question 9 – Solution



Solution

1. Differentiate:

$$x'(t) = -2 \sin 2t, \quad y'(t) = 2 \cos 2t.$$

2. The speed is constant:

$$\sqrt{x'^2 + y'^2} = \sqrt{4 \sin^2 2t + 4 \cos^2 2t} = 2.$$

3. Integrate over the full parameter interval:

$$L = \int_0^{2\pi} 2 \, dt = \boxed{4\pi}.$$

4. The angle of the point is $2t$, which increases from 0 to 4π . Therefore, the unit circle is traversed twice. Its geometric circumference is 2π , but the particle travels that circumference twice, producing total traveled length 4π .