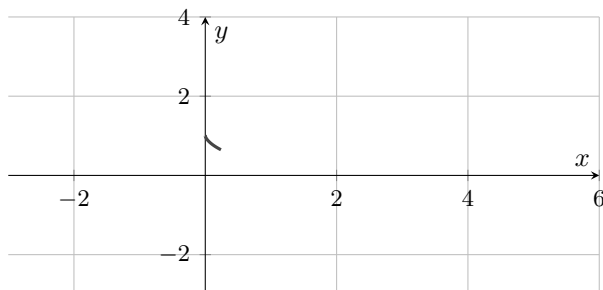


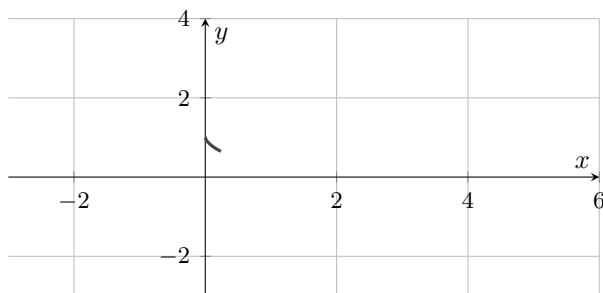
Question 8

Problem

Find the length of the tractable curve $x = t - \tanh t$, $y = \operatorname{sech} t$, $0 \leq t \leq 1$.



Question 8 – Solution



Solution

1. Differentiate and use $1 - \operatorname{sech}^2 t = \tanh^2 t$:

$$x'(t) = 1 - \operatorname{sech}^2 t = \tanh^2 t,$$

$$y'(t) = -\operatorname{sech} t \tanh t.$$

2. Simplify the squared speed:

$$\begin{aligned} x'^2 + y'^2 &= \tanh^4 t + \operatorname{sech}^2 t \tanh^2 t \\ &= \tanh^2 t (\tanh^2 t + \operatorname{sech}^2 t) \\ &= \tanh^2 t. \end{aligned}$$

Thus the speed is $|\tanh t|$.

3. On $0 \leq t \leq 1$, $\tanh t \geq 0$, so

$$L = \int_0^1 \tanh t \, dt.$$

Since $d(\ln \cosh t)/dt = \tanh t$,

$$L = [\ln(\cosh t)]_0^1 = \ln(\cosh 1) - \ln 1.$$

4. Therefore,

$$\boxed{L = \ln(\cosh 1)}.$$