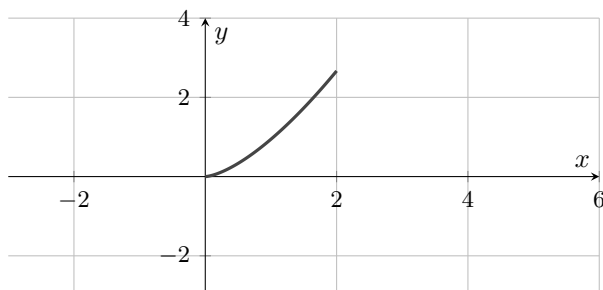


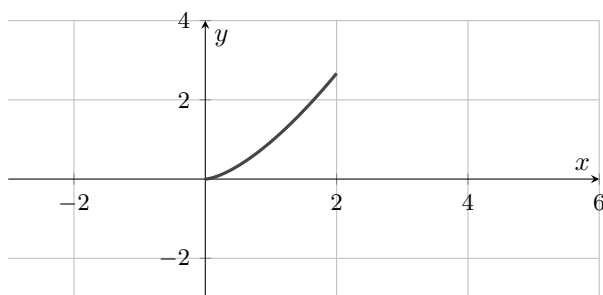
Question 6

Problem

Find the length of $x = t^2/2$, $y = t^3/3$, $0 \leq t \leq 2$.



Question 6 – Solution



Solution

1. Differentiate the coordinates:

$$x'(t) = t, \quad y'(t) = t^2.$$

2. The speed is

$$\sqrt{t^2 + t^4} = \sqrt{t^2(1 + t^2)} = |t|\sqrt{1 + t^2}.$$

Since $0 \leq t \leq 2$, $|t| = t$.

3. Therefore,

$$L = \int_0^2 t\sqrt{1 + t^2} dt.$$

Let $u = 1 + t^2$, so $du = 2t dt$. Then

$$\begin{aligned} L &= \frac{1}{2} \int_1^5 u^{1/2} du \\ &= \frac{1}{3} \left[u^{3/2} \right]_1^5 \\ &= \frac{1}{3} (5^{3/2} - 1). \end{aligned}$$

4. Hence

$$L = \frac{5\sqrt{5} - 1}{3}.$$