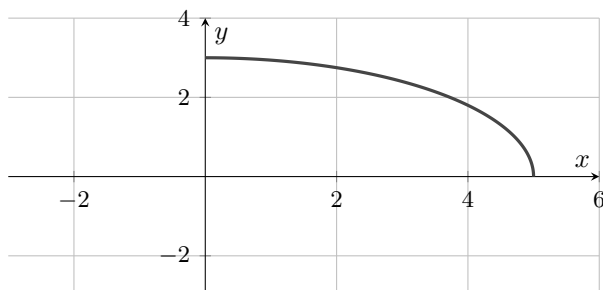


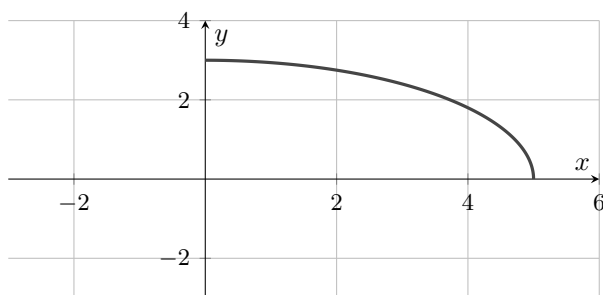
## Question 5

### Problem

A quarter ellipse is  $x = 5 \cos t$ ,  $y = 3 \sin t$ ,  $0 \leq t \leq \pi/2$ . Which simple quarter-circle lengths bound it?



### Question 5 – Solution



### Solution

1. Differentiate:

$$x'(t) = -5 \sin t, \quad y'(t) = 3 \cos t.$$

Thus the speed is

$$v(t) = \sqrt{25 \sin^2 t + 9 \cos^2 t}.$$

2. Rewrite the expression under the square root in two useful ways:

$$25 \sin^2 t + 9 \cos^2 t = 9 + 16 \sin^2 t = 25 - 16 \cos^2 t.$$

Since  $0 \leq \sin^2 t, \cos^2 t \leq 1$ , it follows that

$$9 \leq v(t)^2 \leq 25, \quad \text{so} \quad 3 \leq v(t) \leq 5.$$

3. Integrate these bounds over an interval of width  $\pi/2$ :

$$3 \int_0^{\pi/2} dt \leq L \leq 5 \int_0^{\pi/2} dt.$$

Therefore,

$$\boxed{\frac{3\pi}{2} \leq L \leq \frac{5\pi}{2}}.$$

These are the quarter-circumferences of circles with radii 3 and 5.