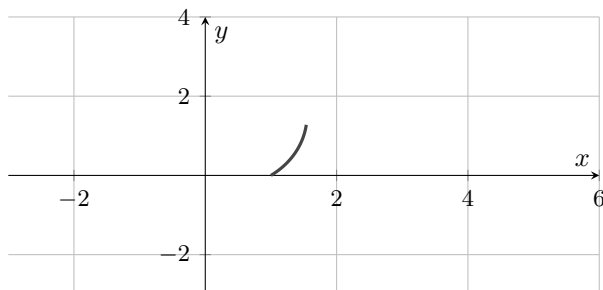


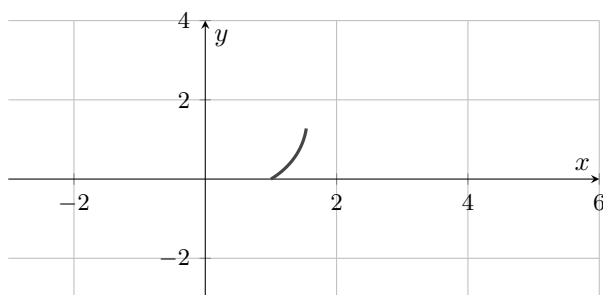
Question 4

Problem

Show that $x = e^t \cos t$, $y = e^t \sin t$, $0 \leq t \leq \ln 2$, has length $\sqrt{2}$.



Question 4 – Solution



Solution

1. Differentiate using the product rule:

$$x'(t) = e^t(\cos t - \sin t), \quad y'(t) = e^t(\sin t + \cos t).$$

2. Compute the squared speed:

$$\begin{aligned} x'^2 + y'^2 &= e^{2t} [(\cos t - \sin t)^2 + (\sin t + \cos t)^2] \\ &= e^{2t} [2 \cos^2 t + 2 \sin^2 t] \\ &= 2e^{2t}. \end{aligned}$$

Since $e^t > 0$, the speed is $\sqrt{2}e^t$.

3. Set up and evaluate the length:

$$\begin{aligned} L &= \int_0^{\ln 2} \sqrt{2}e^t dt \\ &= \sqrt{2} [e^t]_0^{\ln 2} \\ &= \sqrt{2}(2 - 1) = \boxed{\sqrt{2}}. \end{aligned}$$