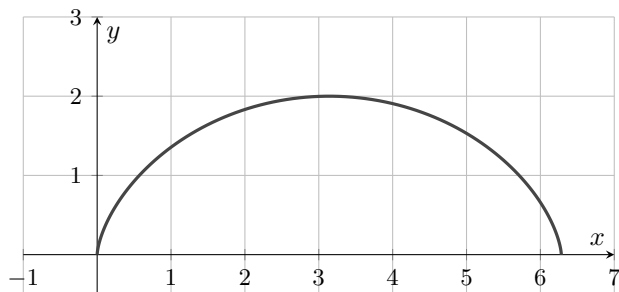


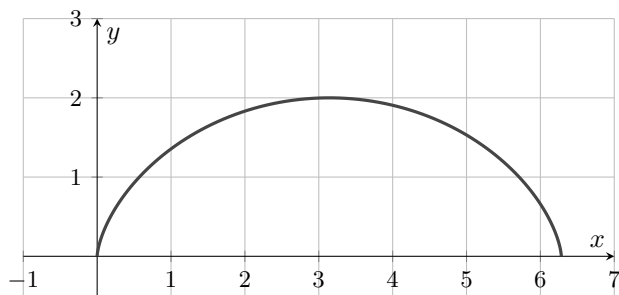
Question 2

Problem

Find the length of one cycloid arch $x = t - \sin t$, $y = 1 - \cos t$, $0 \leq t \leq 2\pi$.



Question 2 – Solution



Solution

1. Differentiate the parametric equations:

$$x'(t) = 1 - \cos t, \quad y'(t) = \sin t.$$

2. Compute and simplify the squared speed:

$$\begin{aligned} x'^2 + y'^2 &= (1 - \cos t)^2 + \sin^2 t \\ &= 1 - 2 \cos t + \cos^2 t + \sin^2 t \\ &= 2 - 2 \cos t = 4 \sin^2 \left(\frac{t}{2} \right). \end{aligned}$$

Hence the speed is

$$2 \left| \sin \left(\frac{t}{2} \right) \right|.$$

3. On $0 \leq t \leq 2\pi$, we have $0 \leq t/2 \leq \pi$, so $\sin(t/2) \geq 0$. Thus

$$L = 2 \int_0^{2\pi} \sin \left(\frac{t}{2} \right) dt.$$

4. Evaluate the integral:

$$L = 2 \left[-2 \cos \left(\frac{t}{2} \right) \right]_0^{2\pi} = -4(\cos \pi - \cos 0) = \boxed{8}.$$