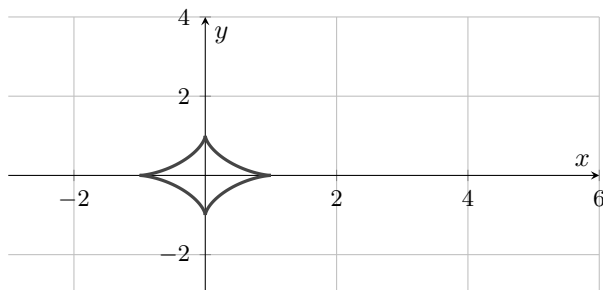


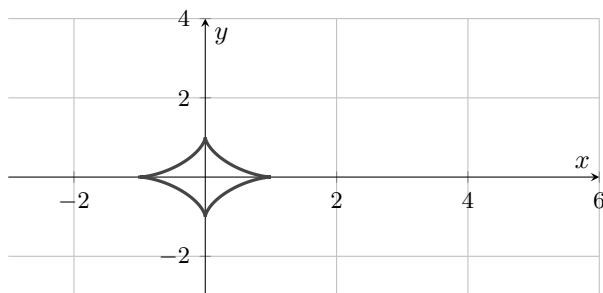
Question 1

Problem

Find the length of the astroid $x = \cos^3 t$, $y = \sin^3 t$.



Question 1 – Solution



Solution

1. The astroid is symmetric about both coordinate axes, so compute the length in the first quadrant, $0 \leq t \leq \pi/2$, and multiply by four.

2. Differentiate:

$$x'(t) = -3 \cos^2 t \sin t, \quad y'(t) = 3 \sin^2 t \cos t.$$

3. Simplify the speed:

$$\begin{aligned} \sqrt{x'^2 + y'^2} &= 3\sqrt{\cos^4 t \sin^2 t + \sin^4 t \cos^2 t} \\ &= 3\sqrt{\sin^2 t \cos^2 t (\cos^2 t + \sin^2 t)} \\ &= 3|\sin t \cos t|. \end{aligned}$$

On $0 \leq t \leq \pi/2$, both factors are nonnegative, so the speed is $3 \sin t \cos t$.

4. The first-quadrant length is

$$L_1 = 3 \int_0^{\pi/2} \sin t \cos t \, dt = \frac{3}{2} [\sin^2 t]_0^{\pi/2} = \frac{3}{2}.$$

5. Therefore, the total length is

$$L = 4L_1 = 4 \left(\frac{3}{2} \right) = 6.$$