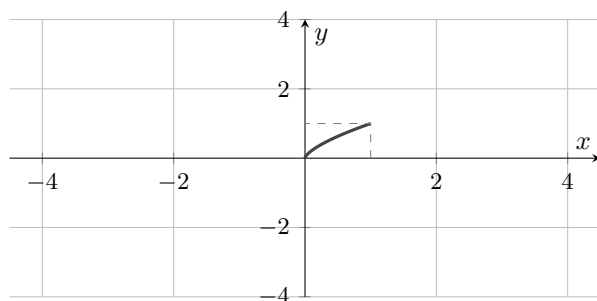


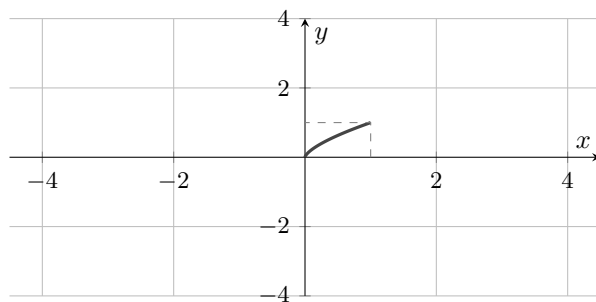
Question 10

Problem

For $x = t^3$, $y = t^2$, $0 \leq t \leq 1$, compare the area under the curve with the area to its left inside the unit square.



Question 10 – Solution

**Solution**

1. As t increases from 0 to 1, the curve runs from $(0, 0)$ to $(1, 1)$ with

$$x'(t) = 3t^2 \geq 0.$$

The area below the curve is therefore

$$A_{\text{below}} = \int_0^1 y(t)x'(t) dt.$$

2. Substitute $y = t^2$ and $x' = 3t^2$:

$$A_{\text{below}} = \int_0^1 3t^4 dt = \left[\frac{3}{5}t^5 \right]_0^1 = \boxed{\frac{3}{5}}.$$

3. The curve divides the unit square into the area below it and the area to its left (equivalently, above it). Since the square has area 1,

$$A_{\text{left}} = 1 - A_{\text{below}} = 1 - \frac{3}{5} = \boxed{\frac{2}{5}}.$$

4. This can also be checked directly with $dy = 2t dt$:

$$A_{\text{left}} = \int_0^1 x dy = \int_0^1 t^3(2t) dt = \frac{2}{5}.$$

Thus the area under the curve is larger by $1/5$.