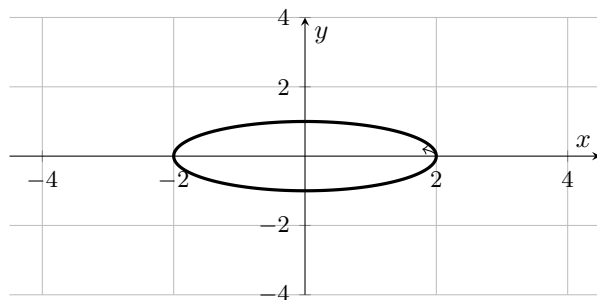
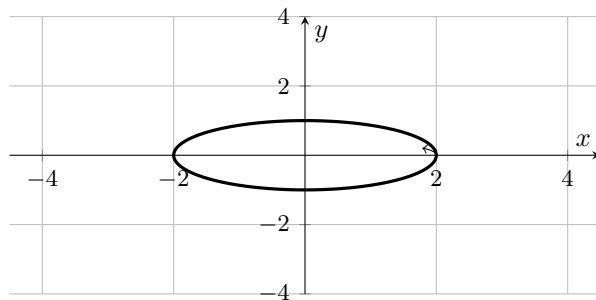


Question 9**Problem**

A designer reverses a parametrization on $0 \leq t \leq 1$ by replacing t with $1 - t$. What happens to the signed area integral $\int yx' dt$?



Question 9 – Solution



Solution

1. Let the original parametrization be $(x(t), y(t))$ on $0 \leq t \leq 1$, with signed integral

$$I = \int_0^1 y(t)x'(t) dt.$$

2. Under the reversed parametrization, set $u = 1 - t$. The new coordinates are

$$\tilde{x}(t) = x(1 - t), \quad \tilde{y}(t) = y(1 - t).$$

By the chain rule,

$$\tilde{x}'(t) = -x'(1 - t).$$

3. The reversed signed integral is

$$\begin{aligned} \tilde{I} &= \int_0^1 \tilde{y}(t)\tilde{x}'(t) dt \\ &= - \int_0^1 y(1 - t)x'(1 - t) dt. \end{aligned}$$

Substituting $u = 1 - t$ shows that $\tilde{I} = -I$.

4. Therefore, reversing the parametrization changes the sign of the signed area integral but not its magnitude:

$$\boxed{\tilde{I} = -I, \quad |\tilde{I}| = |I|}.$$

The geometric area is unchanged.