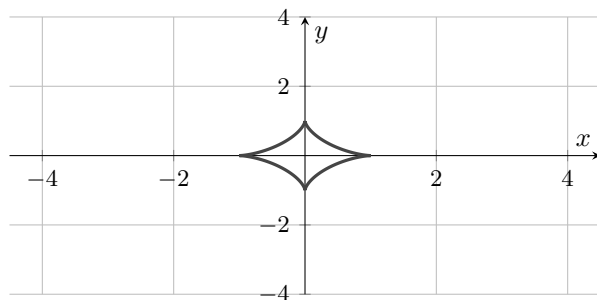


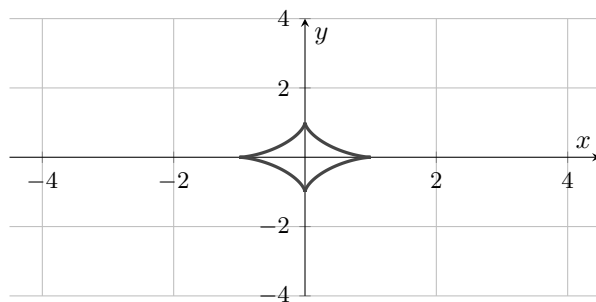
Question 8

Problem

Find the area enclosed by $x = \cos^3 t$, $y = \sin^3 t$ without first eliminating the parameter.



Question 8 – Solution

**Solution**

1. The interval $0 \leq t \leq \pi/2$ traces the first-quadrant arc from $(1, 0)$ to $(0, 1)$. By symmetry, the enclosed area is four times the first-quadrant area.
2. It is convenient to integrate with respect to y :

$$A_1 = \int x \, dy = \int_0^{\pi/2} x(t) y'(t) \, dt.$$

Since $y = \sin^3 t$,

$$y'(t) = 3 \sin^2 t \cos t.$$

3. Substitute $x = \cos^3 t$:

$$A_1 = 3 \int_0^{\pi/2} \sin^2 t \cos^4 t \, dt.$$

Using the even-power integral (or power-reduction identities),

$$\int_0^{\pi/2} \sin^2 t \cos^4 t \, dt = \frac{\pi}{32}.$$

Thus $A_1 = 3\pi/32$.

4. Multiply by four quadrants:

$$A = 4A_1 = 4 \left(\frac{3\pi}{32} \right) = \boxed{\frac{3\pi}{8}}.$$