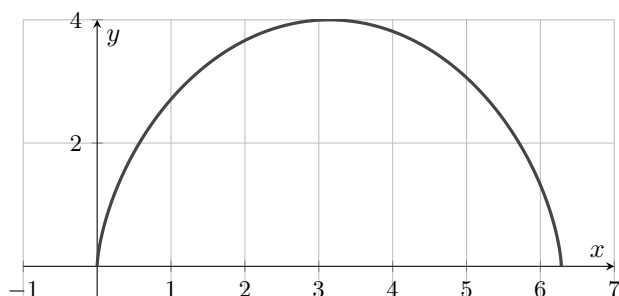


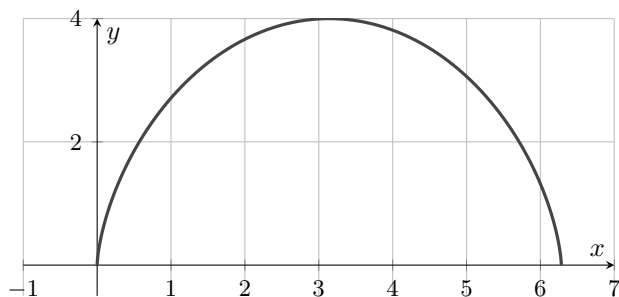
Question 7

Problem

The parametric curve $x = t - \sin t$, $y = 2 - 2 \cos t$ is a stretched cycloid. Predict, then compute, how its arch area compares with the standard cycloid.



Question 7 – Solution



Solution

1. Compared with the standard cycloid $x = t - \sin t$, $y = 1 - \cos t$, the x -coordinates are unchanged while every y -coordinate is multiplied by 2. A vertical stretch by a factor of 2 doubles area, so the predicted arch area is $2(3\pi) = 6\pi$.
2. Verify this prediction directly on $0 \leq t \leq 2\pi$. Differentiate x :

$$x'(t) = 1 - \cos t.$$

Since $x'(t) \geq 0$, the arch is traced from left to right.

3. Substitute $y = 2(1 - \cos t)$:

$$A = \int_0^{2\pi} y(t)x'(t) dt = 2 \int_0^{2\pi} (1 - \cos t)^2 dt.$$

4. From

$$\int_0^{2\pi} (1 - \cos t)^2 dt = 2\pi + \pi = 3\pi,$$

it follows that

$$\boxed{A = 2(3\pi) = 6\pi}.$$

The computation agrees with the scaling prediction.