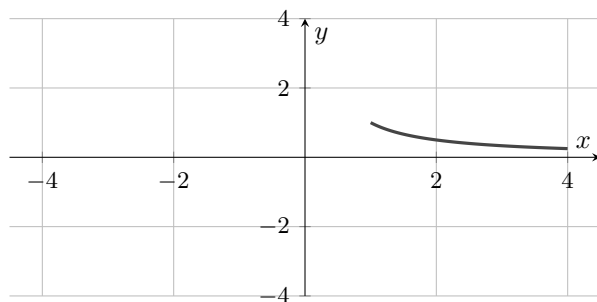


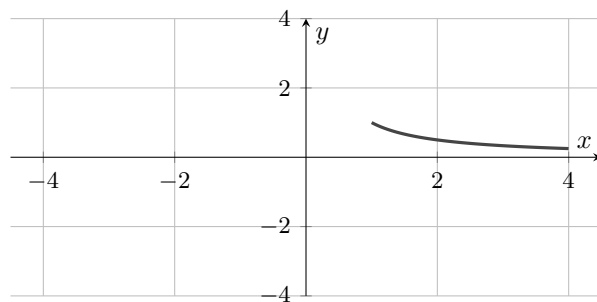
Question 6

Problem

Find the area swept under $x = e^t$, $y = e^{-t}$ from $t = 0$ to $t = \ln 4$.



Question 6 – Solution

**Solution**

1. On $0 \leq t \leq \ln 4$, $x = e^t$ increases from 1 to 4 because

$$x'(t) = e^t > 0.$$

Therefore, the requested area under the curve is directly $\int y \, dx$.

2. Substitute the parametric expressions:

$$A = \int_0^{\ln 4} y(t)x'(t) \, dt = \int_0^{\ln 4} e^{-t} e^t \, dt.$$

3. The exponential factors cancel:

$$e^{-t} e^t = 1.$$

Hence

$$A = \int_0^{\ln 4} 1 \, dt = [t]_0^{\ln 4} = \boxed{\ln 4}.$$

4. As a check, eliminating t gives $y = 1/x$, and $\int_1^4 (1/x) \, dx = \ln 4$, the same result.