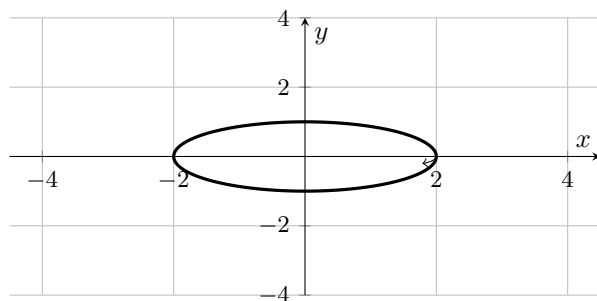


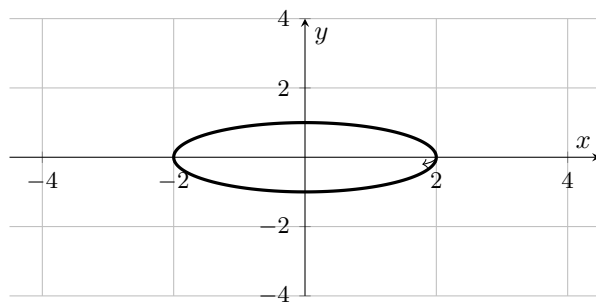
Question 5

Problem

A simple, piecewise smooth closed curve is traversed once clockwise. Explain why $\int y \, dx$ may be positive even though area cannot be negative.



Question 5 – Solution



Solution

1. A line integral around a closed curve is signed: reversing the direction of traversal reverses the sign because dx changes sign.
2. For a positively oriented, counterclockwise curve C , Green's theorem gives

$$A = - \oint_C y \, dx.$$

Thus $\oint_C y \, dx = -A$ for counterclockwise traversal.

3. If the same boundary is traversed clockwise, the line integral changes sign:

$$\oint_{-C} y \, dx = - \oint_C y \, dx = A.$$

Therefore, $\int y \, dx$ can be positive for clockwise motion; the sign describes orientation, not a negative or positive physical area by itself.

4. A direction-independent formula for geometric area is

$$A = \left| \oint_C y \, dx \right|.$$

Equivalently, use $-\oint_C y \, dx$ with counterclockwise orientation or reverse the bounds when necessary.