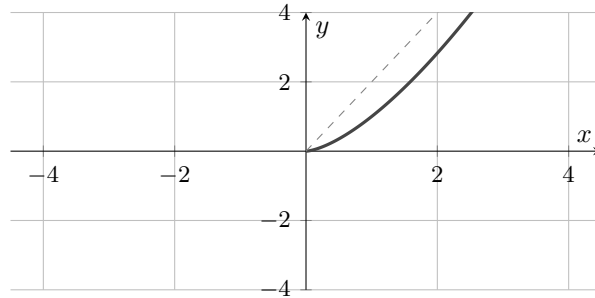
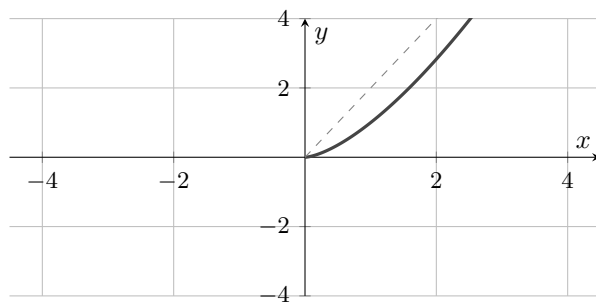


Question 4**Problem**

The curve $x = t^2$, $y = t^3$ joins $(0,0)$ to $(4,8)$. Find the area between it and the chord joining those endpoints.



Question 4 – Solution



Solution

1. The endpoints correspond to $t = 0$ and $t = 2$. The chord through $(0, 0)$ and $(4, 8)$ has slope

$$m = \frac{8 - 0}{4 - 0} = 2,$$

so its equation is $y = 2x$.

2. Along the parametric curve, $x = t^2$ and $y = t^3$. The chord's height at the same x -coordinate is

$$y_{\text{chord}} = 2x = 2t^2.$$

For $0 \leq t \leq 2$, $2t^2 - t^3 = t^2(2 - t) \geq 0$, so the chord lies above the curve.

3. Since $dx/dt = 2t$, the area between them is

$$A = \int_0^2 (y_{\text{chord}} - y_{\text{curve}}) dx = \int_0^2 (2t^2 - t^3)(2t) dt.$$

4. Evaluate:

$$\begin{aligned} A &= \int_0^2 (4t^3 - 2t^4) dt \\ &= \left[t^4 - \frac{2}{5}t^5 \right]_0^2 = 16 - \frac{64}{5} = \boxed{\frac{16}{5}}. \end{aligned}$$