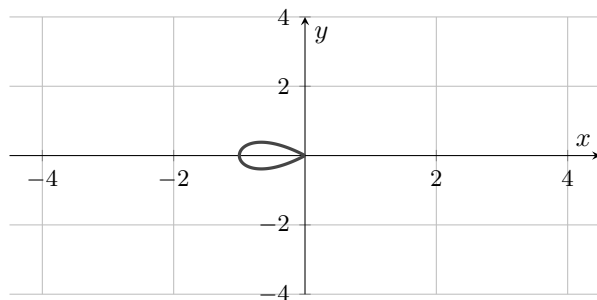
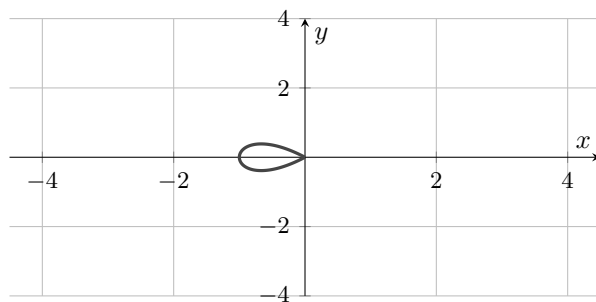


Question 3**Problem**

A loop is traced by $x = t^2 - 1$, $y = t^3 - t$. Find the exact area enclosed by the loop.



Question 3 – Solution



Solution

1. Locate the parameter values at which the loop closes. Since

$$x = t^2 - 1, \quad y = t(t^2 - 1),$$

both $t = -1$ and $t = 1$ produce the point $(0, 0)$. The interval $-1 \leq t \leq 1$ traces the loop once.

2. Differentiate x :

$$x'(t) = 2t.$$

The signed area integral is

$$\int_{-1}^1 y(t)x'(t) dt = \int_{-1}^1 (t^3 - t)(2t) dt.$$

3. Simplify and integrate:

$$\begin{aligned} \int_{-1}^1 (2t^4 - 2t^2) dt &= \left[\frac{2}{5}t^5 - \frac{2}{3}t^3 \right]_{-1}^1 \\ &= \frac{4}{5} - \frac{4}{3} = -\frac{8}{15}. \end{aligned}$$

4. The negative sign records the orientation of traversal; geometric area is nonnegative. Hence

$$A = \left| -\frac{8}{15} \right| = \frac{8}{15}.$$