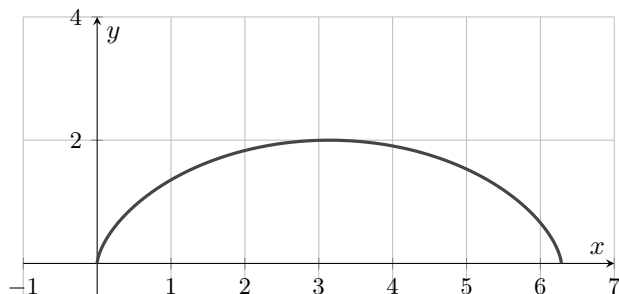


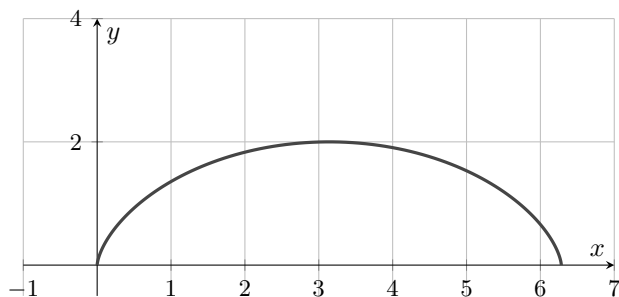
Question 2

Problem

Find the area under one cycloid arch $x = t - \sin t$, $y = 1 - \cos t$, $0 \leq t \leq 2\pi$.



Question 2 – Solution



Solution

1. The given interval traces one arch from $(0,0)$ to $(2\pi,0)$. Since

$$x'(t) = 1 - \cos t \geq 0,$$

the curve moves from left to right, so the area under it is $\int y \, dx$ with the stated bounds.

2. Substitute $y(t) = 1 - \cos t$ and $dx = x'(t)dt$:

$$A = \int_0^{2\pi} y(t)x'(t) \, dt = \int_0^{2\pi} (1 - \cos t)^2 \, dt.$$

3. Expand the integrand:

$$(1 - \cos t)^2 = 1 - 2 \cos t + \cos^2 t.$$

Therefore,

$$A = \int_0^{2\pi} 1 \, dt - 2 \int_0^{2\pi} \cos t \, dt + \int_0^{2\pi} \cos^2 t \, dt.$$

4. These three integrals equal 2π , 0, and π , respectively. Thus

$$\boxed{A = 3\pi}.$$