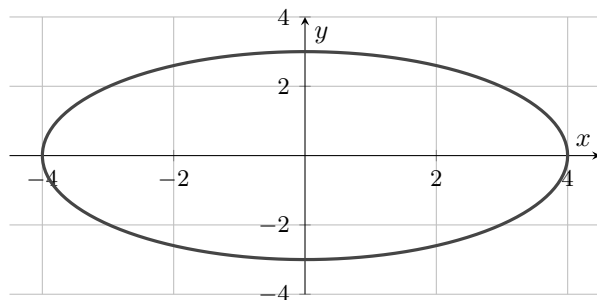
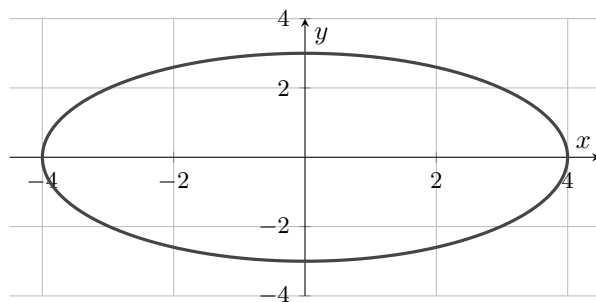


**Question 1****Problem**

Use a parametric integral to find the area of the ellipse  $x = 4 \cos t$ ,  $y = 3 \sin t$ .



## Question 1 – Solution



## Solution

1. The interval  $0 \leq t \leq \pi/2$  traces the ellipse's first-quadrant arc from  $(4, 0)$  to  $(0, 3)$ . By symmetry, the total area is four times the area in that quadrant.

2. Differentiate the horizontal coordinate:

$$\frac{dx}{dt} = -4 \sin t.$$

Because  $x$  decreases on this interval, the first-quadrant area is

$$A_1 = - \int_0^{\pi/2} y(t)x'(t) dt.$$

3. Substitute  $y = 3 \sin t$  and  $x' = -4 \sin t$ :

$$A_1 = - \int_0^{\pi/2} (3 \sin t)(-4 \sin t) dt = 12 \int_0^{\pi/2} \sin^2 t dt.$$

4. Using  $\sin^2 t = (1 - \cos 2t)/2$ ,

$$\int_0^{\pi/2} \sin^2 t dt = \frac{\pi}{4}.$$

Hence  $A_1 = 3\pi$ .

5. Multiply by four quadrants:

$$A = 4A_1 = \boxed{12\pi}.$$