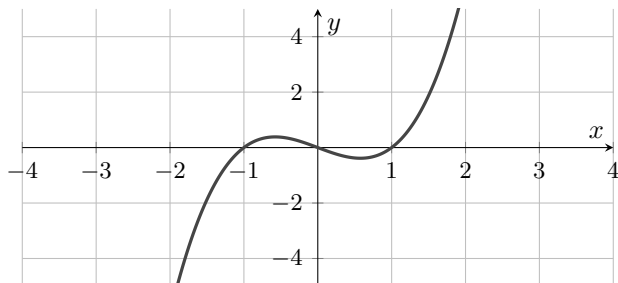


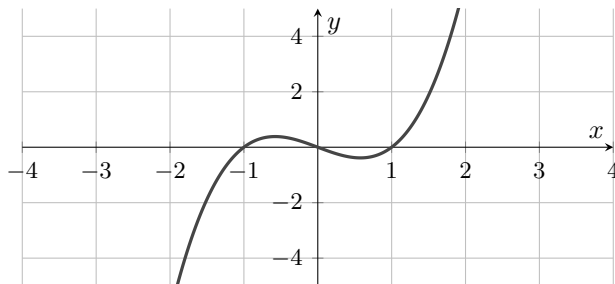
Question 10

Problem

Design a so that $x = t$, $y = t^3 - at$ meets the x -axis at $t = 1$ with slope 2. Is it possible?



Question 10 – Solution



Solution

1. To meet the x -axis at $t = 1$, the y -coordinate must be zero:

$$y(1) = 1^3 - a(1) = 1 - a = 0.$$

Therefore, the position condition requires

$$a = 1.$$

2. Now impose the slope condition. Differentiate:

$$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = 3t^2 - a.$$

Since $dx/dt = 1$, the tangent slope is

$$\frac{dy}{dx} = 3t^2 - a.$$

3. At $t = 1$, requiring slope 2 gives

$$3 - a = 2 \implies a = 1.$$

Thus the position and slope conditions agree.

4. Verify the result directly. With $a = 1$,

$$y(1) = 1 - 1 = 0, \quad \left. \frac{dy}{dx} \right|_{t=1} = 3 - 1 = 2.$$

The point is $(x(1), y(1)) = (1, 0)$, and its tangent line is $y = 2(x - 1)$.

5. Hence the design is possible, and the unique value is

$$\boxed{a = 1}.$$