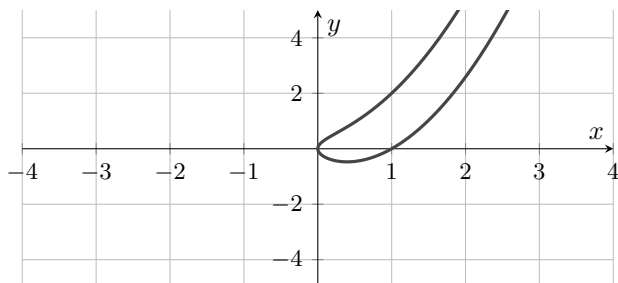


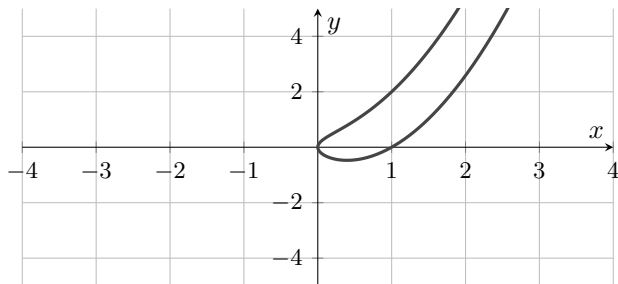
Question 9

Problem

The curve $x = t^2$, $y = t^4 - t$ passes through $(1, 0)$. Find all tangent lines there.



Question 9 – Solution



Solution

1. First find every parameter value that could produce the point $(1, 0)$. From $x = t^2 = 1$,

$$t = 1 \quad \text{or} \quad t = -1.$$

2. Check the y -coordinate for both values:

$$y(1) = 1^4 - 1 = 0, \quad y(-1) = (-1)^4 - (-1) = 2.$$

Only $t = 1$ produces $(1, 0)$, so there can be only one tangent line there.

3. Differentiate:

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 4t^3 - 1.$$

At $t = 1$, $dx/dt = 2 \neq 0$, and the slope is

$$\frac{dy}{dx} = \frac{4t^3 - 1}{2t} = \frac{4(1)^3 - 1}{2(1)} = \frac{3}{2}.$$

4. Apply point-slope form through $(1, 0)$:

$$y - 0 = \frac{3}{2}(x - 1).$$

Therefore, the only tangent line at the specified point is

$$y = \frac{3}{2}(x - 1).$$