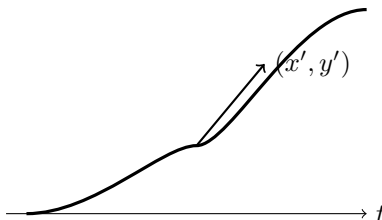
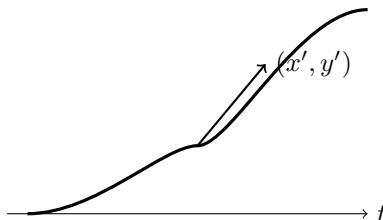


Question 8**Problem**

A particle has velocity $(x', y') = (t - 2, t^2 - 4)$. Does its path have a vertical tangent? Analyze the tangent and direction of motion at any stationary time.



Question 8 – Solution

**Solution**

1. A regular vertical tangent would require

$$x'(t) = t - 2 = 0 \quad \text{and} \quad y'(t) = t^2 - 4 \neq 0.$$

The first condition gives the only candidate, $t = 2$.

2. At $t = 2$,

$$(x'(2), y'(2)) = (0, 0).$$

Thus the particle is momentarily stopped, and the ordinary vertical-tangent test does not apply.

3. For $t \neq 2$, factor y' and simplify the slope:

$$\frac{dy}{dx} = \frac{t^2 - 4}{t - 2} = \frac{(t - 2)(t + 2)}{t - 2} = t + 2.$$

Consequently,

$$\lim_{t \rightarrow 2} \frac{dy}{dx} = 4,$$

which is finite rather than infinite. The path has a tangent of slope 4 at the stationary point, not a vertical tangent.

4. The direction also changes across $t = 2$. Just before 2, both x' and y' are negative, so the particle moves down and left. Just after 2, both are positive, so it moves up and right. At $t = 2$ itself, the velocity is zero and therefore has no instantaneous direction.

5. Hence

the path has no vertical tangent.

The only candidate is a stationary point where the particle reverses direction along a tangent of slope 4.