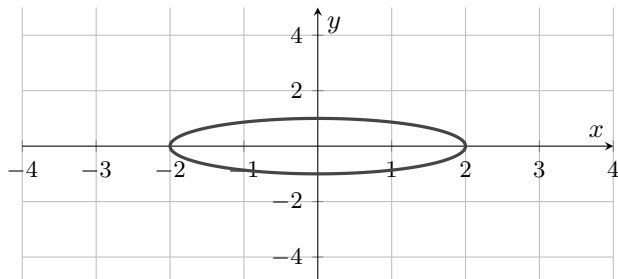
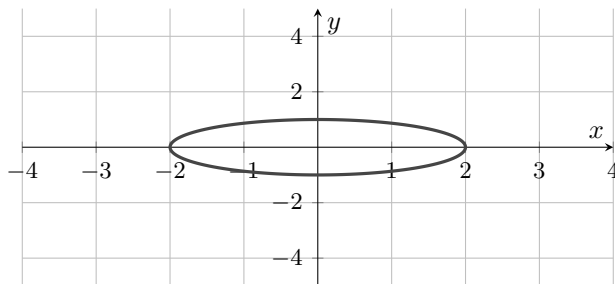


Question 7**Problem**

Find the angle at which the parametric curve $x = 2 \cos t$, $y = \sin t$ crosses the line $y = x/2$ in the first quadrant.



Question 7 – Solution



Solution

1. At an intersection with $y = x/2$, substitute the parametric equations:

$$\sin t = \frac{2 \cos t}{2} = \cos t.$$

In the first quadrant, this gives $t = \pi/4$.

2. The intersection point is

$$\left(2 \cos \frac{\pi}{4}, \sin \frac{\pi}{4}\right) = \left(\sqrt{2}, \frac{\sqrt{2}}{2}\right).$$

3. Differentiate the curve:

$$\frac{dx}{dt} = -2 \sin t, \quad \frac{dy}{dt} = \cos t.$$

Thus its tangent slope at $t = \pi/4$ is

$$m_c = \frac{\cos t}{-2 \sin t} = -\frac{1}{2}.$$

The line $y = x/2$ has slope $m_\ell = 1/2$.

4. If ϕ is the acute angle between two lines, then

$$\tan \phi = \left| \frac{m_\ell - m_c}{1 + m_\ell m_c} \right|.$$

Therefore,

$$\tan \phi = \left| \frac{\frac{1}{2} - (-\frac{1}{2})}{1 + (\frac{1}{2})(-\frac{1}{2})} \right| = \frac{1}{3/4} = \frac{4}{3}.$$

5. Hence the crossing angle is

$$\phi = \arctan \left(\frac{4}{3} \right) \approx 53.13^\circ.$$