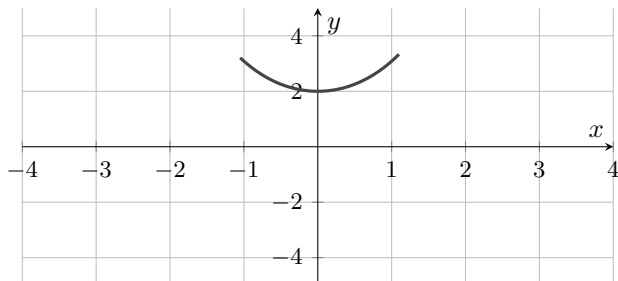
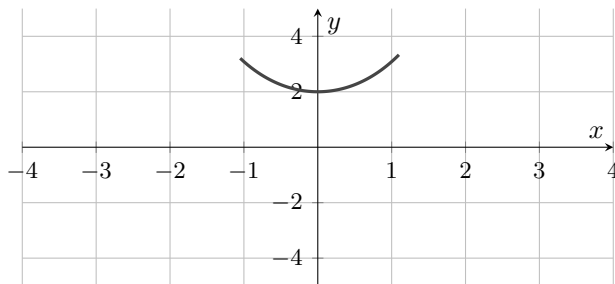


Question 6**Problem**

For $x = \ln t$, $y = t + t^{-1}$, find d^2y/dx^2 at $t = 1$ and classify the local shape.



Question 6 – Solution



Solution

1. Because $x = \ln t$, the parameter domain is $t > 0$. Differentiate the coordinates:

$$\frac{dx}{dt} = \frac{1}{t}, \quad \frac{dy}{dt} = 1 - \frac{1}{t^2}.$$

2. Compute the first derivative with respect to x :

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 - t^{-2}}{t^{-1}} = t - \frac{1}{t}.$$

3. For a parametric curve, differentiate dy/dx with respect to t and divide once more by dx/dt :

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{dx/dt}.$$

Since

$$\frac{d}{dt} \left(t - \frac{1}{t} \right) = 1 + \frac{1}{t^2},$$

we obtain

$$\frac{d^2y}{dx^2} = \frac{1 + t^{-2}}{t^{-1}} = t + \frac{1}{t}.$$

4. Evaluate at $t = 1$:

$$\left. \frac{d^2y}{dx^2} \right|_{t=1} = 1 + 1 = \boxed{2}.$$

5. Because the second derivative is positive, the curve is concave up at this point. Also, $(x(1), y(1)) = (0, 2)$ and $dy/dx = 0$, so $(0, 2)$ is a local minimum of the curve when viewed as y versus x .