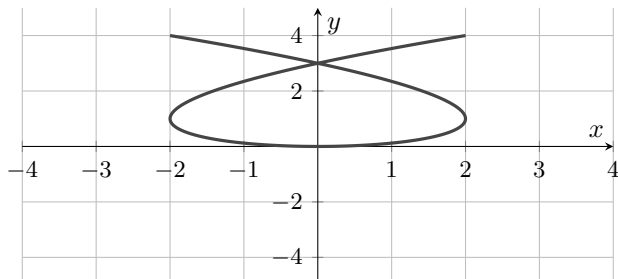


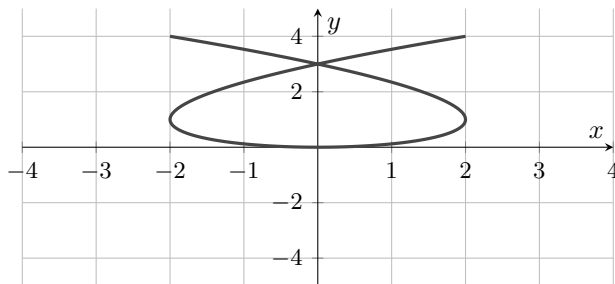
Question 5

Problem

A student claims that the two tangents to $x = t^3 - 3t$, $y = t^2$ at its self-intersection are perpendicular. Locate the crossing and test the claim.



Question 5 – Solution



Solution

- At a self-intersection, two distinct parameter values give the same y -coordinate. Since $y = t^2$, equal y -coordinates occur for parameters t and $-t$.
- Require the corresponding x -coordinates to be equal:

$$t^3 - 3t = (-t)^3 - 3(-t) = -t^3 + 3t.$$

Simplifying gives

$$2t^3 - 6t = 2t(t^2 - 3) = 0.$$

The value $t = 0$ does not give two distinct parameters. The nontrivial pair is

$$t = \pm\sqrt{3}.$$

- At either parameter value,

$$x = t(t^2 - 3) = 0, \quad y = t^2 = 3.$$

Hence the self-intersection is $\boxed{(0, 3)}$.

- Differentiate:

$$\frac{dx}{dt} = 3t^2 - 3, \quad \frac{dy}{dt} = 2t.$$

At $t = \pm\sqrt{3}$, $dx/dt = 6 \neq 0$, so

$$\frac{dy}{dx} = \frac{2t}{3t^2 - 3} = \frac{t}{3}.$$

The two slopes are

$$m_1 = \frac{1}{\sqrt{3}}, \quad m_2 = -\frac{1}{\sqrt{3}}.$$

- Two nonvertical lines are perpendicular only if the product of their slopes is -1 . Here

$$m_1 m_2 = -\frac{1}{3} \neq -1.$$

Therefore, the two tangents are $\boxed{\text{not perpendicular}}$.