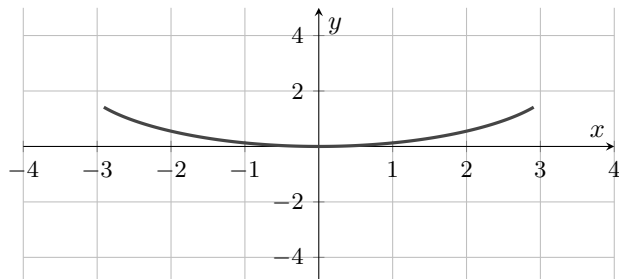


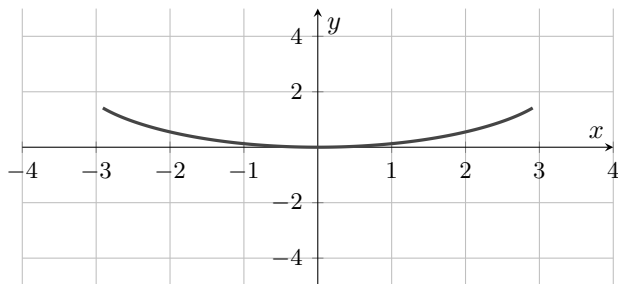
Question 2

Problem

The path $x = t + \sin t$, $y = 1 - \cos t$ models a rolling marker. Find its tangent slope at $t = \pi/2$ and interpret its sign.



Question 2 – Solution



Solution

1. Differentiate the coordinate functions:

$$\frac{dx}{dt} = 1 + \cos t, \quad \frac{dy}{dt} = \sin t.$$

2. At $t = \pi/2$,

$$\frac{dx}{dt} = 1 + \cos \frac{\pi}{2} = 1, \quad \frac{dy}{dt} = \sin \frac{\pi}{2} = 1.$$

Since $dx/dt \neq 0$, the tangent slope is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{1} = \boxed{1}.$$

3. The point on the path is

$$\left(x\left(\frac{\pi}{2}\right), y\left(\frac{\pi}{2}\right)\right) = \left(\frac{\pi}{2} + 1, 1\right).$$

Therefore, the tangent line is

$$y - 1 = x - \left(\frac{\pi}{2} + 1\right).$$

4. Both velocity components are positive at this instant. Thus the marker is moving to the right and upward, which is consistent with the positive slope.