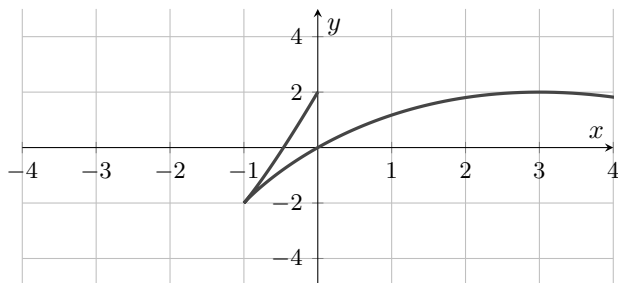


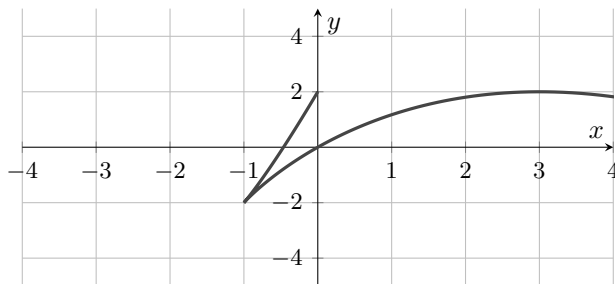
Question 1

Problem

For $x = t^2 - 2t$, $y = t^3 - 3t$, find every horizontal and vertical tangent.



Question 1 – Solution



Solution

1. Differentiate each coordinate:

$$\frac{dx}{dt} = 2t - 2 = 2(t - 1), \quad \frac{dy}{dt} = 3t^2 - 3 = 3(t - 1)(t + 1).$$

2. A horizontal tangent requires $dy/dt = 0$ while $dx/dt \neq 0$. The candidates are

$$3(t - 1)(t + 1) = 0 \implies t = -1, 1.$$

At $t = -1$, $dx/dt = -4 \neq 0$, so this value gives a horizontal tangent. The point is

$$x(-1) = 3, \quad y(-1) = 2.$$

Thus the horizontal tangent line is

$$\boxed{y = 2}.$$

3. At $t = 1$, both derivatives are zero, so the usual horizontal- or vertical-tangent tests are inconclusive. For $t \neq 1$, cancel the common factor:

$$\frac{dy}{dx} = \frac{3(t - 1)(t + 1)}{2(t - 1)} = \frac{3}{2}(t + 1).$$

Therefore,

$$\lim_{t \rightarrow 1} \frac{dy}{dx} = 3.$$

The point at $t = 1$ is $(-1, -2)$, and its tangent line is $y + 2 = 3(x + 1)$; it is neither horizontal nor vertical.

4. A vertical tangent would require $dx/dt = 0$ with $dy/dt \neq 0$. The only zero of dx/dt is $t = 1$, where dy/dt is also zero and the limiting slope is finite. Hence there are no vertical tangents.

5. The complete answer is

$$\boxed{\text{horizontal tangent } y = 2 \text{ at } (3, 2); \quad \text{no vertical tangents.}}$$