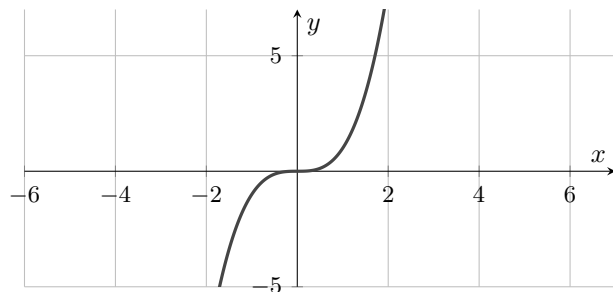


Calculus II Section 3-11: Arc Length and Surface Area Revisited

Question 10

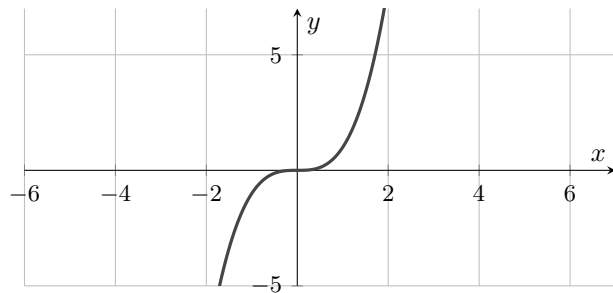
Problem

A numerical system can evaluate either a Cartesian or parametric arc-length integral. Give one reason to prefer each form.



Calculus II Section 3-11: Arc Length and Surface Area Revisited

Question 10 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. Cartesian is convenient for a single-valued smooth graph with simple derivative.
4. Parametric form is preferable for loops, vertical tangents, motion data, or when it avoids an infinite derivative.