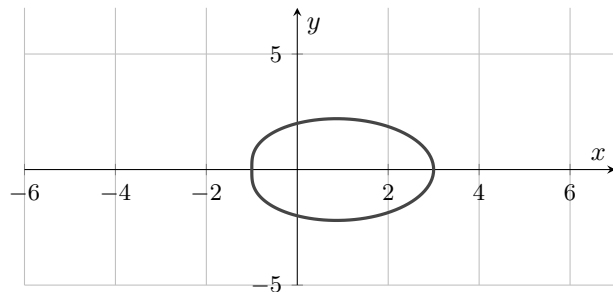


Question 8

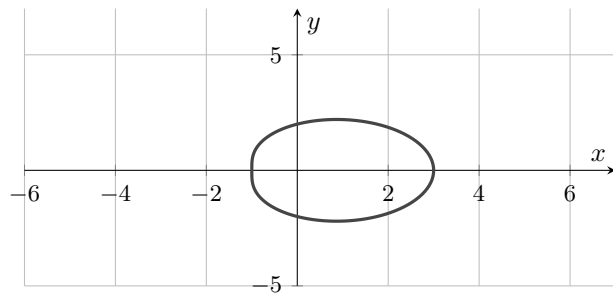
Problem

A curve has polar form $r = f(\theta)$ but is rotated about the y -axis. State when converting to Cartesian form might be safer.



Calculus II Section 3-11: Arc Length and Surface Area Revisited

Question 8 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. Conversion is safer when $x = r \cos \theta$ changes sign or the polar interval retraces points, because the radius to the y -axis is $|x|$ and double-covering is easier to see geometrically.