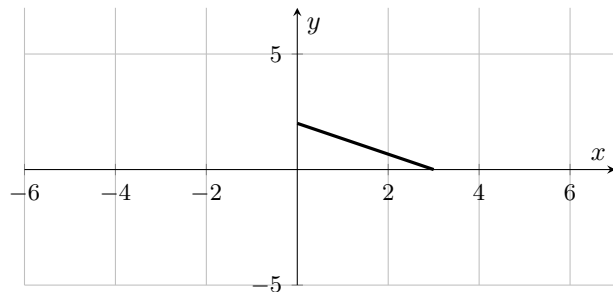


Calculus II Section 3-11: Arc Length and Surface Area Revisited

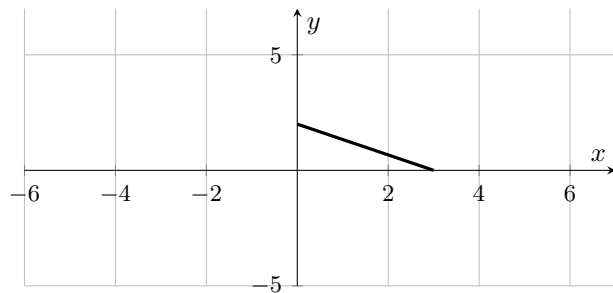
Question 7

Problem

Recover the cone lateral-area formula by parametrizing its slanted edge, then compare it with the slant-height mnemonic.



Question 7 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. For radius $R > 0$ and height $h > 0$, rotate $(x, y) = (Rt/h, h - t)$, $0 \leq t \leq h$, about the y -axis.
4. Then $ds = (\sqrt{R^2 + h^2}/h)dt$ and $S = \pi R\sqrt{R^2 + h^2} = \pi R\ell$, matching the mnemonic.