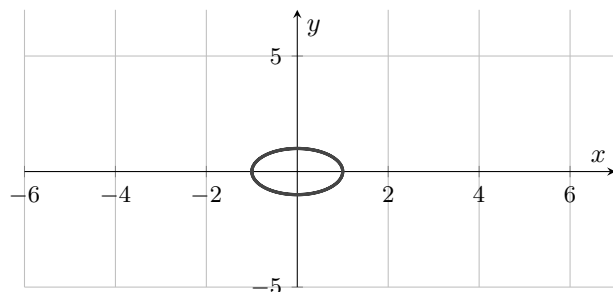


Calculus II Section 3-11: Arc Length and Surface Area Revisited

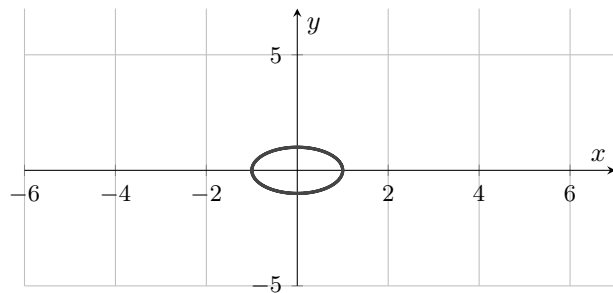
Question 6

Problem

A curve is traced twice by a parametrization but once by a polar interval. Which arc length is the geometric length, and which is the traveled distance?



Question 6 – Solution



Solution

1. Choose a representation and interval that trace the desired curve exactly once. The equivalent arc-length formulas are

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx,$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

2. For a surface of revolution, multiply the appropriate arc-length element by 2π times the nonnegative distance to the axis. Check the tracing interval to prevent geometric double-counting.
3. The once-traced polar integral gives geometric curve length.
4. The twice-traced parametric integral gives total traveled distance, twice the geometric length.